of elementary geometry until the last quarter of the nineteenth century. It was in 1867 and 1868 that Baltzer, Battaglini, Grunert and Hoüel brought Bolyai and Lobatchevsky to the attention of the mathematical public at large.

The new ideas have not affected the teaching of elementary geometry except in some of the definitions and postulates. They have assisted in the rejection of the definitions, "parallel lines are lines everywhere equally distant," and "parallel lines are straight lines which have the same direction." They have shown the futility of "proving" the parallel-postulate and have led to the use of the word "axiom," not as a "self-evident truth," but as a synonym for "postulate."

In conclusion, we note that, with the beginning of the twentieth century, England began once more to influence the teaching of geometry in the United States, through the so-called "Perry movement," and that Germany, which at no time during the nineteenth century affected geometrical teaching in America, makes itself felt at the present time through the pupils of Klein and Hilbert and through the international movement towards reform in the teaching of mathematics, headed by Klein.

ON THE SOLUTIONS OF A SYSTEM OF LINEAR EQUATIONS.

By G. A. MILLER, University of Illinois.

In the June-July number of this journal we proved the theorem that the necessary and sufficient condition that a given unknown has the same value in every solution of a consistent system of linear equations is that the rank of the matrix is reduced by omitting the coefficients of this unknown from this matrix. The following proof of this important theorem is probably more satisfactory to many readers. Suppose that the omission of the coefficients of x_1 reduces the rank of the matrix of the following consistent system of m equations in n unknowns:

$$
\text{(A)}\qquad
\begin{array}{l}
a_{11}\,x_1+a_{12}\,x_2+\ldots+a_{1n}\,x_n+k_1=0,\\
a_{21}\,x_1+a_{22}\,x_2+\ldots+a_{2n}\,x_n+k_2=0,\\
\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\\
\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\\
a_{m1}\,x_1+a_{m2}\,x_2+\ldots+a_{mn}\,x_n+k_n=0.
\end{array}
$$

That is, if r is the rank of the matrix of this system, and if we let

$$a_{i2}\,x_2 + a_{i3}\,x_3 + \ldots + a_{in}\,x_n = f_r, \quad i = 1, 2, \ldots, m,$$

it is possible to find r constants, at least one being different from zero, so that

$$c_1\,f_1 + c_2\,f_2 + \ldots + c_r\,f_r \equiv 0.$$

We may assume that the first r equations of (A) are consistent but not linearly dependent and hence the coefficient of x_1 in the linear binomial equation

$$(c_1\,a_{11} + c_2\,a_{21} + \ldots + c_r\,a_{r1})x_1 + c_1\,k_1 + c_2\,k_2 + \ldots + c_r\,k_r = 0$$

is not zero. This implies that this binomial equation has one and only one solution. That is, x_1 has one and only one value no matter what values of the other unknowns may satisfy the given system of m equations. It may be desirable to mention here another theorem which is, however, more evident. This theorem may be stated as follows:

The necessary and sufficient condition that a given unknown can have the value zero in a consistent system of linear equations is that the matrix of the augmented system must be reduced by omitting the coefficients of this unknown from the system whenever the matrix of the system is reduced by this omission.

In fact, if the omission of these co-efficients does not reduce the rank of the matrix of the system, the unknown in question can have an arbitrary value and hence it can have the value zero. If, on the other hand, this omission does reduce the rank of the matrix as well as that of the augmented matrix, the system will remain consistent, and this unknown can have the value of zero. From the preceding theorem it follows that it can have no other value in this case, and also that it cannot have the value zero when the given omission reduces the rank of the matrix without also reducing the rank of the augmented matrix. That latter fact was suggested to me by Mr. G. Rutledge.

NOTES AND NEWS.

At the University of Chicago Dr. L. E. Dickson has been promoted to a full professorship in mathematics, and Dr. A. C. Lunn to an assistant professorship in applied mathematics. Also Dr. E. T. Wylczynski, of the University of Illinois, has been appointed to an associate professorship in mathematics.